Market impact-aware optimal trading for intraday models

September, 2025

1 Description

1.1 General

For shorter-term models, awareness of costs and market impact becomes increasingly important. One approach to handling this is to let the signal produced by some prediction model be as fast as it needs to be, and then let a market impact model decide upon the optimal trading trajectory. Inserting one's favorite market impact model and formulating this as a continuous-time stochastic optimal control problem, the optimal solution can be obtained through applying e.g. Euler–Lagrange or Hamilton–Jacobi–Bellman. In the general case, a numerical solution can be obtained using some method not suffering from the curse of dimensionality. We follow the presentation in Chapter 11 of Paleologo (2025), which gives an overview of market impact models used alongside portfolio optimization.

1.2 Mathematical details

Let V_t be the wealth at time t, following the dynamics

$$dV_t = \mu_t^{\mathrm{T}} x_t \, dt + x_t^{\mathrm{T}} \sigma_t \, dW_t^1 - c(\dot{x}_t) \, dt.$$

Here, $x_t \in \mathbb{R}^d$ is the position, $W_t^1 \in \mathbb{R}^d$ is a standard Brownian motion, $\mu_t \in \mathbb{R}^d$ is the signal or mean of the returns, $\sigma_t \in \mathbb{R}^{d \times d}$ is the square root of the covariance of the returns, and $c(\dot{x}_t)$ is the instantaneous cost plus market impact at time t. For example, Paleologo (2025) defines this term as

$$c(\dot{x}_t) = \eta^{\mathrm{T}} |\dot{x}_t| + \kappa \int_0^t f(\dot{x}_s) G(t-s) \, ds,$$

decomposing as a transaction cost term plus a market impact term. Common choices for f and G include $f(\dot{x}_t) \propto |\dot{x}_t|^\beta, G(t) = \delta(t)$ (Almgren and Chriss 2000, Almgren *et al.* 2005), $f(\dot{x}_t) \propto \dot{x}_t, G(t) = e^{-\gamma t}$ (Obizhaeva and Wang 2013), and $f(\dot{x}_t) \propto \operatorname{sgn} \dot{x}_t |\dot{x}_t|^{1/2}, G(t) = 1/\sqrt{t}$ (Gatheral 2016). A general setup and solution for certain types of propagators G is discussed in Jaber and Neuman (2022). μ_t can be the signal obtained from some regression model trading too fast to be practically tradable, and $\sigma_t \sigma_t^{\mathrm{T}}$ can be obtained from some standard covariance estimate. In the simplest case, μ_t is deterministic, but it can also follow some dynamics to model some kind of alpha decay, e.g. an Ornstein–Uhlenbeck process

$$d\mu_t = -\alpha \mu_t \, dt + \omega \, dW_t^2.$$

Like Almgren and Chriss (2000) and others, we may optimize some mean–variance objective $\mathbb{E}\, V_T - (\lambda/2)\operatorname{Var} V_T$ for some terminal time T, which may be finite or infinite (Paleologo 2025). A basic example is when μ_t is deterministic and $c(\dot{x}_t) = 0$—in that case we are solving

$$\underset{x}{\text{minimize}} \quad \int_0^T \left(\mu_t^{\mathrm{T}} x_t - \frac{\lambda}{2} x_t^{\mathrm{T}} \Sigma_t x_t \right) dt,$$

where $\Sigma_t = \sigma_t \sigma_t^{\mathrm{T}}$, which just amounts to $x_t \propto \Sigma_t^{-1} \mu_t$ as in Markowitz optimization. Gullberg (2016) and Molnö (2021) are solving a similar problem with $c(\dot{x}_t) = \eta^{\mathrm{T}} |\dot{x}_t|$, which leads to a problem on the form

$$\underset{x}{\text{minimize}} \quad \int_0^T \left(\mu_t^{\mathrm{T}} x_t - \frac{\lambda}{2} x_t^{\mathrm{T}} \Sigma_t x_t - \eta^{\mathrm{T}} |\dot{x}_t|\, dt \right) dt.$$

Qualitatively, in these cases and probably many others involving a nonzero market impact term, a no-trade region occurs—if the partial derivative of the objective with respect to the position is under some threshold in magnitude, no trading is optimal, and otherwise the size of the optimal trade impulse is a function of said derivative. Thus, a problem occurs in which the partial derivative is needed for determining the no-trade region, and the no-trade region is needed for the partial derivative. Molnö (2021) investigates solving this using policy iteration methods used in e.g. reinforcement learning. One can also think of using Feynman–Kac to obtain a backward SDE of which some functional equals our objective, which we can simulate. Various other approaches not suffering from the curse of dimensionality, as e.g. a finite difference method has, are possible.

2 Comments

2.1 Expected results

If successful, the project will result in a deeper understanding of how one can trade based on signals which are too rapidly changing. Being at least in parts coupled with market impact models, it will also increase our knowledge of those and how those affect one's optimal trading trajectory. The project is flexible in the sense that one can start simple and gradually add more advanced market impact and signal decay modeling. Although the methods used in the project may not necessarily be practically useful to a real intraday model, they may inspire certain simplified variants to be used.

2.2 Methodology and technical prerequisites

The project involves a fair amount of theoretical exercise, but also some numerical work (e.g. using PyTorch or other automatic differentiation tools).

2.3 Preferred student profile

A theoretically interested and skilled student, who also has some interest in understanding the mechanisms behind market impact as well as is sufficiently skilled in programming.

References

Almgren R and Chriss N (2000). Optimal execution of portfolio transactions. URL: %5Curl%7Bhttps://www.smallake.kr/wp-content/uploads/2016/03/optliq.pdf%7D.

Almgren R, Thum C and Hauptmann H (2005). Equity market impact. *Risk* 7, pp. 57–62.

Gatheral J (2016). Three models of market impact. URL: %5Curl%7Bhttps://mfe.baruch.cuny.edu/wp-content/uploads/2012/09/Chicago2016OptimalExecution.pdf%7D.

Gullberg D (2016). Optimal trading with transaction costs using a PMP gradient method. MSc thesis. KTH Royal Institute of Technology.

Jaber E and Neuman E (2022). Optimal liquidation with signals: the general propagator case. arXiv: 2211.00447 [q-fin.TR].

Molnö V (2021). Deep learning for dynamic portfolio optimization. MSc thesis. KTH Royal Institute of Technology.

Obizhaeva A and Wang J (2013). Optimal trading strategy and supply/demand dynamics. *Journal of Financial Markets* 16.1, pp. 1–32.

Paleologo G (2025). The elements of quantitative investing. Wiley.